Refusal Under Non-Identifiability
A Recovery-Based Framework for Stability Measurement

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1. Introduction

Stability and resilience metrics are widely used to characterize complex systems in fields such as physiology, ecological monitoring, infrastructure, and machine learning.
In many implementations, stability estimates are emitted even when the underlying dynamics cannot be uniquely inferred from observations.
This creates a structural problem: numerical estimates are produced even when parameters are not identifiable.

Such emission of synthetic stability metrics creates false certainty. In high-stakes systems—medicine, infrastructure monitoring, and automated decision systems—incorrect stability estimates may be more harmful than the absence of a metric.

This paper proposes a recovery-based framework in which stability is inferred only when disturbance–recovery dynamics are observable.

2. Leo-Lo Decomposition of System Dynamics

Let the system state be:

$x(t) \in \mathbb{R}^n$

Leo - instantaneous forcing (memoryless):

$\dot{x}(t) = f(t, x(t)) + \sigma\, \xi(t)$

where:
$f(t, x)$ = structured forcing
$\xi(t)$ = stochastic forcing (noise)
σ = forcing intensity

Lo - causal memory operator:

$Lo(t) = \int_0^\infty K(\tau)\, x(t - \tau)\, d\tau$

where $K(\tau)$ is a causal memory kernel.

Combined LeoLo dynamics:

$\dot{x}(t) = f(t, x(t)) + (Lx)(t) + \sigma\, \xi(t)$

3. Recovery as the Observable of Stability

When forcing ceases and the system returns toward equilibrium, recovery dynamics can be observed.

Under a dominant restoring mode:

$x(t) = A\, e^{(-t/\tau)}$

where τ is the recovery timescale.

The effective stability parameter is:

$k \approx 1 / \tau$

Interpretation:
large k → fast recovery → strong stability
small k → slow recovery → weakening stability

4. Identifiability Boundary

Recovery-based stability estimation requires identifiable recovery dynamics.

Identifiability fails when:

- forcing overlaps the recovery window
- saturation limits the response
- measurement artifacts occur
- forcing termination is unknown
- the observation window is too short

5. Refusal Architecture

When recovery cannot be identified, the system must refuse emission of a stability estimate.

Possible refusal regimes include:

SATURATION – response limited by system capacity
OVERLAPPING_FORCE – forcing continues during recovery
ARTIFACT – corrupted measurements
COLLAPSE_NO_RECOVERY – deviation does not relax

6. Multi-Channel Stability Estimation

Real systems often emit multiple observable signals. Each channel may provide recovery events.

Let $k_c(t_i)$ denote the stability estimate from channel c at event time t_i.

Fusion occurs only across admissible channels:

$k(t_i) \propto \Sigma_{c \in A(t_i)} w_c k_c(t_i)$

where $A(t_i)$ is the set of admissible channels.

7. Theorem - Refusal Under Non-Identifiability

Let $x(t)$ be an observed trajectory after a disturbance.

If recovery cannot be isolated due to overlapping forcing, saturation, artifacts, or insufficient recovery clearance,
then the stability parameter k is not identifiable.

Therefore a stability estimate must not be emitted.
Instead, the system outputs a regime label.

8. Conclusion

Stability is not continuously measurable. It becomes observable only when disturbance–recovery dynamics are identifiable.

The LeoLo framework formalizes this principle and introduces a refusal architecture that prevents emission of stability estimates when recovery cannot be observed.